Reliability of ssGBLUP breeding values in routine test-day multi-trait model

Jiří Bauer*, Josef Přibyl, Luboš Vostrý

Institute of Animal Science, Prague, Czech Republic

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• Routine method for prediction of single-step genomic breeding values developed and running

• Officially published traits: test-day milk production, fat (kg, %), protein (kg, %)

• Reliabilities of GEBVs
Routine evaluation

- Test-day milk production (Zavadilova et al., 2005):
  - **Three traits**: 1st, 2nd, 3rd lactation
- Effects:
  - **Herd-Test-Day** (fixed)
  - Legendre polynomials for lactation*fixed group
    - 4 polynomials per each lactation (fixed)
  - **Regressor for days-in milk** for each lactation (fixed)
  - Legendre polynomials for **additive genetic effect of individual animal**:4 within each lactation with covariance matrix among lactations, i.e. 12x12 elements and connected with relationship matrix (random)
  - Legendre polynomials for **permanent environmental effect** of individual animal:4 within each lactation with covariance matrix among lactations, i.e. 12x12 elements (random)
Routine evaluation

• Single-step GBLUP:

  – **H**: 20% **A**, 80% **G** (Aguilar et al., 2010)

  – **G**:
    • Normalised (aver. diag. = 1) (Forni et al., 2011)
    • Shifted (aver. **G** = aver. **A**<sub>22</sub>) (Vitezica et al., 2011)
Reliability of GEBV - data

- 20,220,047 test-day milk production records of 1,126,102 Czech Holstein cows
- 1,844,679 animals in the pedigree
- 2,236 genotyped bulls; 1,791 with >3 offspring
- Illumina BovineSNP50 Beadchip V2
Reliability of GEBV - method

- Misztal et al. 2013:
  - Approximation of reliability ($R^2$) of ssGBLUP GEBVs
- Strabel et al. 2001:
  - Approximation of $R^2$ in multi-trait evaluation
- Misztal et al. 1991:
  - Approximation of $R^2$ with permanent environmental effect in traits with multiple observations
- Effective number of records (ENR)
Reliability of GEBV - method

• Each trait separately:
  – 1. Approximation of $R^2$ of BLUP EBVs on the basis of effective number of records
  – 2. Calculation of the reliability of genotyped animals (additional genomic information)
  – 3. Addition of the contribution of genotyping to the reliabilities of non-genotyped animals.

• Multi-trait evaluation:
  – 4. Re-evaluation of individual reliabilities due to the covariances among traits (multi-trait model)
Reliability of GEBV – BLUP

1. Approximation of $R^2$ of BLUP EBVs on the basis of effective number of records

- Calculation of ENR for each animal and iterative procedure for taking into account the relationships of animals (Misztal et al., 1993)

- The effective number of records $d$ for each animal was updated by the formula: $d^* = \tau* d / (\tau + d)$ (Misztal et al., 1991) where $\tau$ is the ratio of residual variance to permanent environmental variance
Reliability of GEBV – ssGBLUP

- 2. Calculation of the reliability of genotyped animals (additional genomic information)

  - Reliability \( r^2_i = 1 - \alpha q_{ii} \)

\( \alpha \) - ratio of error variance to animal genetic variance
\( q_{ii} \) - the diagonal elements of the \( Q^{-1} \) matrix:

\[
Q^{-1} = [D + (I + G^{-1} - A_{22}^{-1}) \alpha]^{-1}
\]

\( D \) - contribution of the records and pedigrees to the reliability from step (1)
\( I \) - identity matrix
\( G^{-1} \) - inverse of the genomic relationship matrix
\( A_{22}^{-1} \) - inverse of the section of the pedigree-based relationship matrix that contains relationship information from only the genotyped animals

(Misztal et al., 2013)
• 3. Addition of the contribution of genotyping to the reliabilities of non-genotyped animals.

  – Procedure similar to the step (1) of the method

  – The reliabilities of the genotyped bulls kept constant:
    • to avoid double counting the contribution of the relationships among these bulls
    • to avoid changing the already fully conveyed values.
• 4. Re-evaluation of individual reliabilities due to the covariances among traits (multi-trait model)

- Taking into account the additive genetic covariances among traits

- \( r_i^2 = 1 - (\hat{\mathbf{C}}^{-1}) / (\mathbf{G}_0) \)

\( \mathbf{G}_0 \) - variance covariance matrix of the additive genetic effect

\( \hat{\mathbf{C}}^{-1} \) - inversion of the approximation of the left-hand side of the mixed-model equations for the animal and its progeny

(Strabel et al., 2001)
Reliability of GEBV - method

• Three lactation index for routine publication:

  \[- \text{Reliability} = \frac{(b'Pb)}{(a'G_Ta)}\]

  \(b\) - vector of weights in index for three lactations
  \(P\) - covariance matrix of breeding values
  \(a\) - vector of weights of genetic values (1/3 each trait)
  \(G_T\) - genetic covariance matrix among lactations
## Reliability of GEBV - results

<table>
<thead>
<tr>
<th>R² values for:</th>
<th>Step of method</th>
<th>No. of animals</th>
<th>1st lactation mean</th>
<th>1st lactation s.d.</th>
<th>2nd lactation mean</th>
<th>2nd lactation s.d.</th>
<th>3rd lactation mean</th>
<th>3rd lactation s.d.</th>
<th>3-lactation index mean</th>
<th>3-lactation index s.d.</th>
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<tbody>
<tr>
<td>Whole population</td>
<td>BLUP</td>
<td>1,854,275</td>
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<td>.339</td>
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<td>.157</td>
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</tbody>
</table>
Reliability of GEBV - results

Number of genotyped bulls by reliability of EBV/GEBV in different parts of procedure: 1st Lactation
GEBVs and reliabilities are continually published at [http://www.plemdat.cz](http://www.plemdat.cz)
Reliability of GEBV - plans

• Compare the approximate reliabilities with predictions of GEBVs given the truncated data

• Compare the approximated reliability values with results of simulations
Reliability of GEBV

Thank you for your attention